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ABSTRACT

The purpose of this study was to determine the relative effectiveness of the College Entrance Examination Board (CEEB) Mathematics Achievement Test, Level I, as an instrument for placing students in beginning college mathematics courses. For this purppse it was compared with several other variables as predictors of success in the specific courses; passing was considered success in a course. The other variables gathered were: number of high school mathematics units; high school class rank (HSR); and the verbal, mathematics (SAT-M) and total scores of the CEEB Scholastic Aptitude Tests. The sample consisted of students in one of seven different beginning mathematics courses offered by Indiana University. Based on several multiple regression analyses, it was concluded that the combination of SAT-M and HSR was the most efficacious predictor of success. More specific results for particular courses and different combinations of predicotr variables are reported with complete statistical results provided in the appendix of this document. (JP)

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An Analysis of Achievement Behavior in Selected Mathematics Courses

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INDIANA STUDIES IN PREDICTION No. 12

AN ANALYSIS OF ACHIEVEMENT BEHAVIOR IN SELECTED MATHEMATICS COURSES

by

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Monograph of the

Bureau of Educational Studies
and Testing

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PREFACE

Educators are constantly seeking means to predict the future behavior of students. We wish to make these predictions as efficiently as possible, capitalizing first on student data currently available. This monograph, the twelfth in the series, was prepared to answer placement questions posed by the Mathematics Department and the Junior Division at Indiana University.

Perhaps the most significant outcome of this study is the presentation of expectancy tables which can be used by advisors who have minimal qualifications in interpretation of statistical data. We urge that the results of the use of the tables be made available to the Bureau for further study and refinement.

The Bureau of Educational Studies and Testing is a university-wide applied research and evaluation service center. One of our major objectives has been to conduct a series of studies in the prediction of student behavior. The first eleven monographs by title are:

No. 1 "Predicting Success for University Freshmen"

No. 2 "Predicting Individual Course Success for Entering Freshmen"

No. 3 "The Prediction and Analysis of Grade Achievement Behavior"

No. 4 "Predicting Success for Advanced Graduate Students in Education"

No. 5 "Predicting Success for Master's Degree Students in Education"

No. 6 "The University Freshman Dropout"

No. 7 "Who Shall Persist?"

No. 8 "Tests of English Language as Predictors of Success for Foreign Students"

No. 9 "An Analysis of Achievement Behavior in the Law School"

No. 10 "Modern Language Placement: An Alternative to Testing"

No. 11 "Freshmen View The College Scene: Opinions Before and After the Initial Semester"

Many persons have helped in the work of the rather extensive research effort. Chief among these have been Dr. George Springer, Chairman, Department of Mathematics; and Mr. Donald Brineman, Assistant Dean of Junior Division. Also, we sincerely appreciated the cooperation of the students who participated in the study.

R.C.P.

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INDIANA STUDIES IN PREDICTION: No. 12

AN ANALYSIS OF ACHIEVEMENT BEHAVIOR IN SELECTED MATHEMATICS COURSES

Background

During the academic year of 1967-68, the Mathematics Department and the Junior Division at Indiana University jointly requested that the Bureau of Educational Studies and Testing conduct a criterion-related validity study of the College Entrance Examination Board (CEEB) Mathematics Achievement Test, Level I, for the beginning mathematics courses at Indiana University.

An ever present problem for counselors and advisors is the correct placement of incoming students in courses so that the student is challenged and yet can attain an acceptable degree of success. This monograph, number twelve in the series of prediction studies at Indiana University, addresses itself to this problem in order to provide information leading to better placement of students in beginning mathematics courses. The criterion of success in this project was defined as the final grade received in the respective mathematics courses.[1]

Predictor variables considered in this study were scores on the CEEB Mathematics Achievement Test, Level I, and the verbal, mathematics, and total scores of the CEEB Scholastic Aptitude Test. Other predictor variables were the number of units of high school mathematics (Units) which was defined as the number of years during which a student was enrolled in mathematics courses and the high school rank (HSR) which is defined as the student's rank in graduation class divided by the class size.

High school rank was scaled in terms of deciles. The point which defines the top ten percent of a given graduating high school class is called the first decile. Any student who is in the top ten percent of his graduating class is said to be in the first decoid[2] where a decoid is the interval between any two deciles.

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[1]A weighting system was used in which an "A" was given a value of 4, "B" a value of 3, "C" a value of 2, "D" a value of 1, and "F" a value of 0.

[2]Chase, C. I., Elementary Statistical Procedures, McGraw Hill, 1967.

Similarly, the point which defines the next ten percent of a given graduating class is called the second decile. Therefore, a student who ranks in this part of his class is said to be in the second decoid.

Purposes

The specific purposes of this project at Indiana University were: 1) to determine the relative effectiveness of the Mathematics Achievement Test, Level I, as an instrument for placing students in beginning mathematics courses; 2) to determine the most effective combination of various predictors of achievement in beginning mathematics courses; and 3) to develop expectancy tables for predicting success in the beginning mathematics courses as an aid to advisors in placing students. The purposes were essentially served by conducting three consecutive studies during the academic year 1967-68.

Descriptions and Procedures

The mathematics courses included in this project are described in the following paragraphs and will thereafter be referred to by the course number accompanying the description.[3]

A Review of Algebra and Trigonometry (M115) is a five credit hour course which does not count toward a degree in the College of Arts and Sciences and is described by its title.

A Partial Review of Algebra and Trigonometry (M117) is a three credit hour course which is also described by its title and does not count toward a degree in the College of Arts and Sciences.

Finite Mathematics (M118) is a three credit hour course concerned with set theory, linear systems, matrices, determinants, probability and linear programming with applications to problems in the social sciences.

A Brief Survey of Calculus (M119) is a three credit hour course providing an introduction to calculus primarily for students in the social sciences.

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[3]Course descriptions are taken from the 1968-69 College of Arts and Sciences Bulletin, Indiana University.

Plane Analytic Geometry (M131) is a two credit hour course dealing with coordinate systems, loci, equations of curves, straight line, circle, conic sections and general second degree equations.

Analytic Geometry and Calculus I-II (M215-216) are sequential five credit hour courses dealing with coordinates, functions, straight line, limits, continuity, derivative and definite integrals, applications, circles, conics, techniques of integration and infinite series.

The instruments used in this project are described in the following paragraphs and are thereafter referred to by the abbreviations which accompany the descriptions.

The Scholastic Aptitude Test[4] (SAT) of the College Entrance Examination Board is a three hour objective test designed to measure the development of verbal and mathematical skills and yields three scores. A verbal score (SAT V) is the measure of reading comprehension and word-idea relationships. A mathematical score (SAT M) is the measure of problem solving through understanding of and reasoning with mathematical symbols. The third score sometimes used as a general indicator of aptitude is the arithmetic total of SAT V and SAT M, (SAT TOT).

The intended purpose of the test is the prediction of individual academic performance which may be applied to admissions or placement functions.

The Mathematics Achievement Test, Level I[5] (LEVEL I) of the College Entrance Examination Board is a one hour objective test. It is a broad range, cumulative test with approximately half the focus of the test on algebra and plane geometry. The remainder deals with such topics as coordinate geometry, trigonometry, functions, space perceptions and simple solids, mathematical reasoning and the nature of proof. The purpose of the test is "to measure the level of achievement in particular subject fields"[6] which for this test is mathematics.

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[4] A Description of the College Board Scholastic Aptitude Test, College Entrance Examination Board, 1967-68.

[5] A Description of the College Board Scholastic Aptitude Test, College Entrance Examination Board, 1968-69.

[6] Bulletin of Information College Board Admissions Test, College Entrance Examination Board, 1968-69, p. 4.

The first study of the project was conducted in the fall semester of the academic year, 1967-68. The sample consisted of all students who received a final grade in M115, M117 and M215 and for whom information pertaining to LEVEL I, SAT V, SAT M scores as well as HSR and Units was available. Students receiving grades of W (withdrawn) or I (incomplete) and those utilizing the pass-fail option were not included in the study. Students receiving a grade of W-F (withdrawn failing) were included with those students receiving a final grade of F (failing). The preceding information was taken from student files and final grades were obtained from final grade reports.

Although there were small Ns, the trends in the means of the variables were particularly interesting as all means progrers in the expected direction. The one possible exception is that of the average grade which is lower in M215 than in the other two courses. Means and standard deviations were generated for all variables, distributed by courses and are reported along with the respective sample sizes in Table 1.

TABLE 1. MEANS AND STANDARD DEVIATIONS OF COURSE GRADE, CEEB SCHOLASTIC APTITUDE TEST MATHEMATICS AND VERBAL SCORES, HIGH SCHOOL RANK, CEEB MATHEMATICS ACHIEVEMENT TEST LEVEL I, AND UNITS OF HIGH SCHOOL MATHEMATICS DISTRIBUTED BY MATHEMATICS COURSE, STUDY I.

Variable		Course M115 N = 20	M117 N = 59	M215 N = 83
GRADE	M	2.75	2.76	2.40
	SD	1.02	1.28	1.11
SAT V	M	423.50	493.25	516.64
	SD	78.29	88.81	80.02
SAT M	M	512.50	560.00	626.31
	SD	62.06	73.95	65.73
HSR	M	3.70	2.40	1.80
	SD	2.40	1.70	1.50
LEVEL I	M	487.00	531.36	609.04
	SD	60.36	72.65	71.86
UNITS	M	3.10	3.36	3.68
	SD	.72	.58	.49

A limitation of the results in this first study resulted from the general lack of complete sets of predictive measures on sufficient sample sizes, particularly for LEVEL I scores. As a result of the small sample sizes in the first study it was decided to conduct a second study to provide more information relative to the LEVEL I test.

Since it was apparent that few additional subjects with LEVEL I scores as a predictive measure would become available, the LEVEL I test was administered in conjunction with final examinations concluding the spring semester. Two parallel forms of the LEVEL I test were provided by the Educational Testing Service for this purpose. An important distinction between this study and the previous one is that LEVEL I is a "concurrent" measure rather than a predictive one.

Data for this study consisted of HSR, SAT V, SAT M collected from student records; the final course grade collected from final grade reports; and the LEVEL I score from the end-of-course administration. Samples for this study consisted of those students enrolled in M115 and M117 during the spring semester of the academic year 1967-68, and for whom complete data were available. Again, students utilizing the pass-fail option or receiving grades of I or W were excluded from the study. Those receiving grades of W-F were included and were pooled with those receiving a grade of F. A possible source of bias in this respect is the exclusion from the study of those students who as a result of impending failure in the course did not take the final examination and consequently the LEVEL I test.

Again, the same trend in means of the variables was evident. All means progress in the expected direction with the exception of the mean grade for the students in M117 which is lower than that of M115. Means and standard deviations for each variable within a given course are presented in Table 2.

TABLE 2. MEANS AND STANDARD DEVIATIONS OF COURSE GRADE, CEEB SCHOLASTIC APTITUDE TEST MATHEMATICS AND VERBAL SCORES, HIGH SCHOOL RANK AND CEEB MATHEMATICS ACHIEVEMENT TEST LEVEL I DISTRIBUTED BY MATHEMATICS COURSE, STUDY II.

Variable		Course M115 N = 211	M117 N = 154
GRADE	M	2.59	2.31
	SD	1.11	1.22
SAT V	M	484.10	496.32
	SD	95.14	89.12
SAT M	M	493.30	548.87
	SD	95.52	78.81
HSR	M	3.20	2.70
	SD	1.90	2.00
LEVEL I	M	503.76	529.77
	SD	78.36	70.47

The third and final study of this project was conducted as a result of decisions made on the basis of the two previous studies.

The samples consisted of all students enrolled in M115, M117, M118, M119, M131, M215 and M216 during the academic year 1967-68 for whom SAT V, SAT M, SAT TOT, HSR and final course grade could be obtained and, in part, overlaps with the samples in the first two studies. Exclusions from the study were the same as for the previous studies and students receiving grades of W-F were pooled with those receiving grades of F.

Trends in the means were generally similar to the trends in the previous studies. One rather consistent exception to these trends was M119 in which student performance was very high.

Means and standard deviations classified by course are presented for each variable in Table 3.

TABLE 3. MEANS AND STANDARD DEVIATIONS OF COURSE GRADE, CEEB SCHOLASTIC APTITUDE TEST VERBAL, MATHEMATICS AND TOTAL SCORES, AND HIGH SCHOOL RANK DISTRIBUTED BY MATHEMATICS COURSE, STUDY III.

Variable		Course M115 N = 402	M117 N = 412	M118 N = 297	M119 N = 149	M131 N = 107	M215 N = 448	M216 N = 238
GRADE	M	2.49	2.35	2.35	2.60	2.39	2.36	2.49
	SD	1.18	1.22	1.11	1.12	1.14	1.19	1.18
SAT V	M	476.35	496.73	502.17	537.21	516.37	520.71	538.30
	SD	93.27	88.59	89.26	98.20	87.04	87.62	99.13
SAT M	M	496.48	553.98	579.50	601.39	580.20	618.64	631.23
	SD	89.13	76.16	79.58	81.26	72.41	73.82	77.40
SAT TOT	M	972.83	1050.71	1081.67	1138.60	1096.57	1139.34	1169.53
	SD	157.03	140.43	143.15	155.61	135.87	139.10	156.69
HSR	M	3.20	2.70	2.60	1.90	2.40	2.00	1.70
	SD	2.00	1.90	1.70	1.30	1.80	1.60	1.30

These data were used to generate expectancy tables and multiple linear regression equations for predicting success in the various mathematics courses included in this project. Tables and equations will be presented in a later section of this monograph.

Results

In each of the studies further analysis consisted of zero-order intercorrelations between all variables within a given study distributed by course. In addition a stepwise multiple linear regression technique was employed to generate multiple linear regression equations and multiple correlation coefficients for predicting achievement in the respective courses from a weighted combination of the various predictor variables.

In a stepwise multiple regression approach the variable selected at the first step is the one which makes the greatest reduction in the residual sum of squares. At each successive step a variable is selected in such a manner as to provide the greatest reduction in the residual sum of squares when taken in combination with the subset of variables selected by the previous step.

An alternative to allowing the variables to enter the regression equation in a stepwise manner is to control their entry by specifying the order of entry. In study I, all predictor variables were allowed to enter the regression equation in a stepwise manner except LEVEL I, which was suppressed until the last step. The effect was to provide information regarding the increase in predictive power due to the addition of LEVEL I, over all other variables.

STUDY I

The sample for M115 consisted of only 20 Ss which was considered too small for regression analysis. The sample for M117 consisted of 59 Ss. It can be noted that SAT M correlated the highest with the criterion followed by LEVEL I, HSR, Units and SAT V respectively. (Table 4)

When one considers the summary regression table in which all predictor variables were allowed to enter in a stepwise manner (Appendix A), it is observed that the order in which the predictor

variables entered the equation was SAT M, Units, HSR, LEVEL I and finally SAT V. This may be accounted for in part by the correlation of .58 between SAT M and LEVEL I.

The multiple correlation (R) between all predictor variables and the criterion is .72 while the value for SAT M, Units and HSR as predictors of GRADE is .69. The variables SAT V and LEVEL I apparently add little to the predictive power. This is further evident when one considers the reduction in the standard error of estimate of .95 for SAT M, Units and HSR as opposed to .93 for all variables (see Appendix A).

By forcing LEVEL I into the equation last, it is seen that very little is added to the predictive power. This is seen by the small increase in the multiple correlation and the small reduction in the standard error of estimate (Appendix A).

The multiple linear prediction equation for M117 using all variables is presented in Appendix B.

The sample for M215 contained 89 Ss. All correlations between the predictors and the criterion were low with LEVEL I having the highest correlation of .35 (see Table 4). Interestingly enough when allowed to enter the regression equation in a stepwise manner, LEVEL I entered first while SAT M was the last to enter. The order in which the intervening variables entered was SAT V, HSR, and Units.

The fact that SAT M was the last to enter may be partially explained by the intercorrelation of .64 between LEVEL I and SAT M. The relative order of HSR and SAT V remains the same as for M117; however, Units is of a lower priority. This is not totally unexpected since placement in the various courses is contingent upon Units. For students placed in M215 the distribution of Units has a more restricted range of possible values than for M115 or M117 (see Tables 1, 2 and 3).

The fact that LEVEL I enters the equation first indicates that it is the most effective in reducing the residual sum of squares. This is supported by forcing LEVEL I to enter the equation last and noticing the 6 percent increase of criterion variance accounted for and the accompanying, although relatively small, reduction in the standard error of estimate.

The question is now raised as to the efficacy of LEVEL I as opposed to a weighted combination of SAT M and HSR. When LEVEL I is allowed to enter the equation first, the correlation is .35 which is significant at the .01 level of significance. LEVEL I accounts for 12 percent of the criterion variance and when HSR, SAT V, Units and SAT M are allowed to enter the equation

first they account for an additional 2 percent of the criterion variance. The corresponding decrease in the standard error of estimate is negligible (Appendix A). Thus, LEVEL I alone provides as much information as all variables taken together. If no information regarding LEVEL I were available (simulated by forcing LEVEL I to enter the regression equation last) it is noted that SAT M enters the equation first followed by HSR. These two variables taken together produce a multiple R of .28 which is significant at the .05 level with a standard error of estimate of 1.08. These two variables account for slightly more than 8 percent of the criterion variance. It can further be seen that SAT V and Units add little to the predictive power of this multiple regression equation. Therefore, LEVEL I alone accounts for 12 percent of the criterion variance with an accompanying standard error of estimate of 1.05 which is an increase of 4 percent over the 8 percent of accounted-for variance produced by the weighted combination of SAT M and HSR. The .03 error in estimated grade advantage for LEVEL I (1.08-1.05) over SAT M and HSR is considered to be negligible.

In summary, the information obtained served as a preliminary indictment against LEVEL I on the basis of the small return realized from the cost in time and money necessary to collect the data.

The multiple prediction equation using all variables is presented in Appendix B.

TABLE 4. ZERO ORDER INTERCORRELATIONS BETWEEN ALL VARIABLES AND MULTIPLE CORRELATION COEFFICIENTS BETWEEN ALL PREDICTOR VARIABLES AND GRADE DISTRIBUTED BY MATHEMATICS COURSE, STUDY I.

Variables	SAT V	SAT M	HSR	LEVEL I	UNITS
M117 (N = 59)					
GRADE	.33	.60	-.41	.52	.34
SAT V		.31	-.25	.27	-.06
SAT M			-.28	.58	.13
HSR				-.18	-.27
LEVEL I					.26
MULTIPLE CORRELATION = .72					
M215 (N = 83)					
GRADE	.08	.23	-.23	.35	-.08
SAT V		.44	-.35	.48	-.11
SAT M			-.34	.64	-.17
HSR				-.45	.22
LEVEL I					-.04
MULTIPLE CORRELATION = .38					

STUDY II

The data were not available to analyze further LEVEL I as a predictor variable. The only workable way for analysis of LEVEL I as a predictor of GRADE was to administer LEVEL I as a posttest to students enrolled in each mathematics course. However, in order to gain some insight into the relationship between LEVEL I, SAT M and GRADE, LEVEL I was administered in conjunction with final examinations in M115 and M117 during the spring semester.

The sample for M115 consisted of 211 Ss for whom complete data were available. In this study LEVEL I was administered as a concurrent validity measure rather than a predictive measure.

The zero order correlations of SAT M and LEVEL I with the criterion were .46 and .45 respectively, followed by HSR and SAT V. A most revealing correlation is the one between SAT M, a predictive measure and LEVEL I, a concurrent measure, the magnitude of which is .71 (see Table 5). Since LEVEL I is not a predictive measure, only SAT M, HSR and SAT V were allowed to enter the multiple regression equation and they entered in that order. The multiple correlation using all three predictors was .56 with a standard error of estimate of .93. If only SAT M and HSR are considered, the multiple correlation is .55 with a standard error of estimate of .94. It is, therefore, obvious that SAT V adds little to the predictive power.

The multiple regression equation using SAT M and HSR is presented in Appendix B.

In the data for M117 the zero order correlations between the respective variables and the criterion lacked the general consistency heretofore exhibited. HSR with a correlation of .57 (disregarding the negative sign) was highest followed by LEVEL I, SAT V and SAT M (see Table 5). However, consistency was seen in the relatively high correlation between SAT M and LEVEL I, .61.

HSR was selected first by the stepwise routine in multiple regression followed by SAT M which increased the correlation (R) to .59. When SAT V was included, the multiple R was .60. Although HSR produced the greatest single reduction in residual score variance, consistency was exhibited in that SAT V added little to the predictive power.

The multiple linear regression equation for predicting GRADE from SAT V, SAT M and HSR is presented in Appendix B.

M215 is the first semester course of a two course sequence and hence no data were available for M215 in this study during the spring semester.

TABLE 5. ZERO ORDER INTERCORRELATIONS BETWEEN ALL VARIABLES AND MULTIPLE CORRELATION COEFFICIENTS BETWEEN ALL PREDICTOR VARIABLES* AND GRADE DISTRIBUTED BY MATHEMATICS COURSE, STUDY II.

Variable	SAT V	SAT M	HSR	LEVEL I
M115 (N = 211)				
GRADE	.17	.46	-.39	.45
SAT V		.44	-.33	.30
SAT M			-.20	.71
HSR				.23
	MULTIPLE CORRELATION = .56			
M117 (N = 154)				
GRADE	.37	.36	-.57	.50
SAT V		.50	-.41	.41
SAT M			-.38	.61
HSR				.36
	MULTIPLE CORRELATION = .60			

*LEVEL I was not a predictor variable and was not included in the multiple correlation.

In view of the consistently high correlation between SAT M and LEVEL I scores, the fact that for LEVEL I to be a predictive measure it would have to be administered as a pretest to those enrolled in each course and that SAT M and HSR were consistently the best predictors of GRADE to the exclusion of the SAT V, it was decided that approximately as many error free placement decisions could be made using predictive measures SAT M and HSR. Exclusion of SAT V from the multiple prediction equation is supported by the consistently high intercorrelations between SAT V and SAT M, the subsequent small increase in the multiple R and the corresponding small reduction in the standard error of estimate.

STUDY III

In study III the course enrollments varied from 107 in M131 to 448 in M215 (see Table 3). The data for these seven courses were relatively consistent. In making contrasts, multiple correlations using SAT TOT and HSR as predictors were compared to those using SAT V, SAT M and HSR to determine which combination of variables would be the best predictor of GRADE. Criteria used to make this determination were criterion variance accounted for by the weighted combinations of the variables and the reduction of the standard error of estimate.

In each case the multiple correlation obtained by using SAT V, SAT M and HSR was as great as that obtained by using SAT TOT and HSR. Further, the standard error of estimate obtained by using SAT M, SAT V and HSR was approximately the same as that obtained by using SAT TOT and HSR (see Table 6). One notices that in every case where the multiple correlation is obtained by using SAT V, SAT M and HSR, SAT V is the last to enter the equation and never adds more than one percent of accounted for criterion variance. If at all, it only slightly reduces the standard error of estimate.

Another interesting finding from these data was the moderately high intercorrelation between SAT M and SAT V (median r = .48). This corroborates the evidence presented regarding its selection for the regression equations.

In courses M115, M118 and M119, SAT M entered the regression equation first, followed by HSR. In courses M117, M131, M215 and M216 the order of entry was reversed. Examination of similarities of course descriptions failed to explain this reversal.

TABLE 6. ZERO ORDER INTERCORRELATIONS BETWEEN ALL VARIABLES AND MULTIPLE CORRELATION COEFFICIENTS BETWEEN A WEIGHTED COMBINATION OF HIGH SCHOOL RANK AND CEEB SCHOLASTIC APTITUDE MATHEMATICS SCORE AS PREDICTORS OF GRADE DISTRIBUTED BY MATHEMATICS COURSE, STUDY III.

Variable	GRADE	SAT V	SAT M	SAT TOT	HSR
M115 (N = 402)					
GRADE	1.00	.26	.49	.43	-.44
SAT V		1.00	.48	.87	-.31
SAT M			1.00	.85	-.25
SAT TOT				1.00	.33
HSR					1.00
	MULTIPLE CORRELATION = .59				
M117 (N = 412)					
GRADE	1.00	.27	.40	.39	-.47
SAT V		1.00	.45	.88	-.33
SAT M			1.00	.83	-.30
SAT TOT				1.00	-.37
HSR					1.00
	MULTIPLE CORRELATION = .54				
M118 (N = 297)					
GRADE	1.00	.26	.39	.38	-.36
SAT V		1.00	.44	.87	-.34
SAT M			1.00	.83	-.27
SAT TOT				1.00	-.36
HSR					1.00
	MULTIPLE CORRELATION = .47				
M119 (N = 149)					
GRADE	1.00	.20	.27	.27	-.22
SAT V		1.00	.50	.89	-.42
SAT M			1.00	.84	-.21
SAT TOT				1.00	-.38
HSR					1.00
	MULTIPLE CORRELATION = .31				

TABLE 6 (Continued)

Variable	GRADE	SAT V	SAT M	SAT TOT	HSR
M131 (N = 107)					
GRADE	1.00	.21	.33	.31	-.53
SAT V		1.00	.45	.88	-.32
SAT M			1.00	.83	-.06
SAT TOT				1.00	-.23
HSR					1.00
MULTIPLE CORRELATION = .61					
M215 (N = 448)					
GRADE	1.00	.25	.33	.33	-.36
SAT V		1.00	.48	.89	-.31
SAT M			1.00	.83	-.32
SAT TOT				1.00	-.37
HSR					1.00
MULTIPLE CORRELATION = .42					
M216 (N = 238)					
GRADE	1.00	.14	.14	.16	-.24
SAT V		1.00	.57	.91	-.34
SAT M			1.00	.85	-.30
SAT TOT				1.00	-.36
HSR					1.00
MULTIPLE CORRELATION = .25					

Expectancy tables for predicting success as measured by course grade for each course contingent upon SAT M and HSR have been constructed on the basis of actual performance by those students involved in this study. These tables are presented for each course in Appendix C. Multiple linear regression equations using HSR and SAT M as predictors of GRADE are presented for each course in Appendix B.

Summary and Conclusions

The Bureau of Educational Studies and Testing was requested by the Mathematics Department and the Junior Division at Indiana University to conduct a criterion related validity study of achievement in beginning mathematics courses during the academic year of 1967-68. Preliminary investigation revealed that the correlation between SAT M and LEVEL I, both predictive measures, was high. However, some ambiguity was introduced as LEVEL I contributed little to the predictive power of GRADE over a weighted combination of SAT M and HSR in M117 but entered the regression equation first in M215.

In M215, LEVEL I alone accounted for 12 percent of the criterion variance with a correlation of .35, significant at the .01 level. SAT M and HSR, with a multiple R of .28, significant at the .05 level accounted for 8 percent of the criterion variance. The difference of .03 of an estimated grade between the standard error of estimate was considered of little consequence. The 4 percent advantage of LEVEL I over the weighted combination of SAT M and HSR hardly seemed to justify the expense in time and money which would have to be incurred due to administrating the LEVEL I test to all students enrolling in mathematics. This was especially true when one considers the ambiguity revealed by the data from M117.

In Study II more information was sought concerning LEVEL I before a decision concerning its validity could be reached. Consequently, it was administered in conjunction with final examination in M115 and M117 in the spring semester of the academic year 1967-68. The correlations between SAT M as a predictive measure and LEVEL I as a concurrent measure of .71 and .61 for M115 and M117 respectively tended to corroborate the decision to rely on SAT M and HSR rather than LEVEL I for decision making purposes.

On the basis of these decisions attention was focused on SAT V, SAT M, SAT TOT and HSR as predictors of achievement in courses M115, M117, M118, M119, M131, M215 and M216.

All subjects enrolled in these courses during the academic year 1967-68 for whom complete data were available with those exceptions noted in an earlier part of the study were included in the analysis.

In each case, SAT M and HSR were as effective as SAT TOT and HSR on the basis of the multiple correlation and the standard error of estimate. Further, in no case did SAT V add more than

1 percent to the accounted for criterion variance nor decrease the standard error of multiple estimate by more than .01 of a grade unit. Hence, it was decided that the most efficacious combination of predictors of achievement in beginning mathematics courses was SAT M and HSR. The order in which these two variables entered the multiple regression equation was SAT M and HSR respectively for M115, M118 and M119 and the reverse for M117, M131, M215 and M216.

Prediction equations and expectancy tables for predicting success based on SAT M and HSR were generated for each course and are presented in Appendix B and C respectively.

APPENDIX A

APPENDIX A

Appendix A provides summary tables of the stepwise multiple regression technique for each course within each study. This information is presented in order that the reader may examine the rationale behind the decisions made in this study.

The step number indicates the order in which the variables were selected. Each variable is selected in such a way that when combined with the variables of the previous step the greatest reduction in the residual sum of squares occurs.

The multiple R is an index of the relationship between the predictors and the criterion. The square of the multiple R gives the proportion of criterion variance accounted for by that combination of predictors. The increase in RSQ allows one to assess the contribution of the last variable in terms of accounted for variance.

The standard error of estimate allows one to establish a band about the predicted grade within which the actual grade of a certain percent of students should fall.

SUMMARY TABLES OF STEPWISE MULTIPLE REGRESSION PROCEDURE CLASSIFIED BY COURSE WITHIN STUDIES.

STUDY I

TABLE 7. ALL PREDICTOR VARIABLES WERE ALLOWED TO ENTER THE EQUATION IN A STEPWISE MANNER.

STEP NUMBER	VARIABLE ENTERED	R	RSQ	INCREASE IN RSQ	S.E. ESTIMATE
M117 (N = 59)					
1	SAT M	.60	.36	.36	1.03
2	UNITS	.65	.42	.06	.98
3	HSR	.69	.48	.06	.95
4	LEVEL I	.71	.50	.02	.94
5	SAT V	.72	.52	.02	.93
M215 (N = 83)					
1	LEVEL I	.35	.12	.12	1.05
2	SAT V	.36	.13	.01	1.05
3	HSR	.37	.14	.01	1.05
4	UNITS	.38	.14	.00	1.06
5	SAT M	.38	.14	.00	1.06

TABLE 8. PREDICTOR VARIABLES SAT V, SAT M, UNITS AND HSR WERE ALLOWED TO ENTER IN A STEPWISE MANNER. LEVEL I WAS FORCED INTO THE EQUATION LAST.

STEP NUMBER	VARIABLE ENTERED	R	RSQ	INCREASE IN RSQ	S.E. ESTIMATE
M117 (N = 59)					
1	SAT M	.60	.36	.36	1.03
2	UNITS	.65	.42	.06	.98
3	HSR	.69	.48	.06	.95
4	SAT V	.70	.49	.01	.95
5	LEVEL I	.72	.52	.03	.93
M215 (N = 83)					
1	SAT M	.23	.05	.05	1.09
2	HSR	.28	.08	.03	1.08
3	SAT V	.29	.08	.01	1.09
4	UNITS	.29	.08	.00	1.09
5	LEVEL I	.38	.14	.06	1.07

STUDY II

TABLE 9. ALL PREDICTOR VARIABLES WERE ALLOWED TO ENTER THE PREDICTION EQUATION IN A STEPWISE MANNER.

STEP NUMBER	VARIABLE ENTERED	R	RSQ	INCREASE IN RSQ	S.E. ESTIMATE
M115 (N = 211)					
1	SAT M	.46	.21	.21	1.00
2	HSR	.55	.30	.09	.94
3	SAT V	.56	.31	.01	.93
M117 (N = 154)					
1	HSR	.57	.32	.32	1.01
2	SAT M	.59	.35	.03	.99
3	SAT V	.60	.36	.01	.99

STUDY III

TABLE 10. ALL PREDICTOR VARIABLES WERE ALLOWED TO ENTER THE PREDICTION EQUATION IN A STEPWISE MANNER.

STEP NUMBER	VARIABLE ENTERED	R	RSQ	INCREASE IN RSQ	S.E. ESTIMATE
M115 (N = 402)					
1	SAT M	.49	.24	.24	1.03
2	HSR	.59	.35	.11	.95
3.	SAT V	.59	.35	.00	.95
M117 (N = 412)					
1	HSR	.47	.22	.22	1.08
2	SAT M	.54	.29	.07	1.03
3	SAT V	.55	.30	.01	1.03
M118 (N = 297)					
1	SAT M	.39	.15	.15	1.02
2	HSR	.47	.22	.07	.98
3	SAT V	.47	.22	.00	.98
M 119 (N = 149)					
1	SAT M	.27	.07	.07	1.08
2	HSR	.31	.10	.03	1.07
3	SAT V	.32	.10	.00	1.07
M131 (N = 107)					
1	HSR	.53	.28	.28	.97
2	SAT M	.61	.37	.09	.92
3	SAT V	.61	.37	.00	.91
M215 (N = 448)					
1	HSR	.36	.13	.13	1.11
2	SAT M	.42	.18	.05	1.08
3	SAT V	.43	.18	.00	1.08

TABLE 10 (Continued)

STEP NUMBER	VARIABLE ENTERED	R	RSQ	INCREASE IN RSQ	S.E. ESTIMATE
M216 (N = 238)					
1	HSR	.23	.05	.05	1.14
2	SAT M	.25	.06	.01	1.14
3	SAT V	.25	.06	.00	1.14

APPENDIX B

APPENDIX B

In Appendix B are presented the regression equations for predicting the final grade in a given mathematics course using mathematics scores on the SAT and high school rank expressed as a decoid. For example, let us predict the grade of Isaac in M131 when his SAT M score is 400 and his HSR is 3.2. His predicted grade is then found by completing the following computation: PREDICTED GRADE equals .00465 (400) - .32449 (3.2) + .47446. This value when computed is 1.30 to two significant figures. By adding and subtracting the standard error of estimate of .92 a predicted grade range from .38 to 2.32 is obtained. Approximately two-thirds of the students who have a weighted combination of SAT M scores and HSR equal to Johnny's will have actual grades within this range.

STUDY I

TABLE 11. MULTIPLE LINEAR REGRESSION EQUATIONS FOR PREDICTING COURSE GRADE.

Equation	Standard error of estimate
1. M117 (N = 59) .00179(SAT V) + .00634(SAT M) + .00331(LEVEL I) + .45743(UNITS) - .14728(HSR) - 4.60678 = PREDICTED GRADE	.93
2. M215 (N = 83) -.00194(SAT V) + .00046(SAT M) + .00537(LEVEL I) - .11885(UNITS) - .07839(HSR) + .42046 = PREDICTED GRADE	1.06

STUDY II

TABLE 12. MULTIPLE LINEAR REGRESSION EQUATIONS FOR PREDICTING COURSE GRADE.

Equation	Standard error of estimate
1. M115 (N = 211)	
-.00162(SAT V) + .00519(SAT M) -	
.20113(HSR) + 1.45841 = PREDICTED GRADE	.93
2. M117 (N = 154)	
.00145(SAT V) + .00201(SAT M) -	
.29677(HSR) + 1.31265 = PREDICTED GRADE	.99

STUDY III

TABLE 13. MULTIPLE LINEAR REGRESSION EQUATIONS FOR PREDICTING COURSE GRADE.

Equation	Standard error of estimate
1. M115 (N = 402)	
.00526(SAT M) - .20545(HSR) + .54792 = PREDICTED GRADE	.95
2. M117 (N = 412)	
.00456(SAT M) - .24756(HSR) + .51153 = PREDICTED GRADE	1.03
3. M118 (N = 297)	
.00445(SAT M) - .18063(HSR) + .25343 = PREDICTED GRADE	.98
4. M119 (N = 149)	
.00316(SAT M) - .14417(HSR) + .98533 = PREDICTED GRADE	1.07
5. M131 (N = 107)	
.00465(SAT M) - .32449(HSR) + .47446 = PREDICTED GRADE	.92
6. M215 (N = 448)	
.00388(SAT M) - .20949(HSR) + .39675 = PREDICTED GRADE	1.08
7. M216 (N = 238)	
.00122(SAT M) - .18941(HSR) + 2.05473 = PREDICTED GRADE	1.14

APPENDIX C

APPENDIX C

Appendix C presents expectancy tables based on SAT M scores and HSR for each course.

These tables show frequencies, cumulative frequencies and cumulative percents of students whose SAT M scores fall within a given range and whose HSR expressed as a decoid fall within a given range that achieve a grade of A, B, C, D or F in a given mathematics course.

For example, of the thirty-nine students in M115 whose SAT M scores were from 500 to 549 and whose HSR was in the first or second decoid, seventeen students, or forty-four percent received A's. Twenty-six of the thirty-nine, or sixty-seven percent received A's or B's while virtually one hundred percent of those students received grades of A, B, or C.

TABLE 14. EXPECTANCY TABLE FROM STUDY III FOR FINAL GRADE IN MATHEMATICS M115 BASED ON CEEB SCHOLASTIC APTITUDE TEST MATHEMATICS SCORE AND HIGH SCHOOL RANK (N = 402).

HIGH SCHOOL RANK	FINAL GRADE	SAT MATH 200-449 f	cf	cp	450-499 f	cf	cp	500-549 f	cf	cp	550-800 f	cf	cp
	A	7	7	22	6	6	22	17	17	44	38	38	54
	B	7	14	44	9	15	56	9	26	67	24	62	87
1-2	C	10	24	75	10	25	93	13	39	100	7	69	97
	D	5	29	91	1	26	96	0	39	100	1	70	99
	F	3	32	100	1	27	100	0	39	100	1	71	100
	A	3	3	8	3	3	10	6	6	22	12	12	44
	B	6	9	24	8	11	38	8	14	52	6	18	67
3-4	C	17	26	68	8	19	66	11	25	93	8	26	96
	D	9	35	92	7	26	90	2	27	100	1	27	100
	F	3	38	100	3	29	100	0	27	100	0	27	100
	A	0	0	0	2	2	6	2	2	13	3	3	15
	B	2	2	5	8	10	28	11	13	87	5	8	40
5-10	C	15	17	41	12	22	61	1	14	93	6	14	70
	D	18	35	85	9	31	86	0	14	93	6	20	100
	F	6	41	100	5	36	100	1	15	100	0	20	100

TABLE 15. EXPECTANCY TABLE FROM STUDY III FOR FINAL GRADE IN MATHEMATICS M117 BASED ON CEEB SCHOLASTIC APTITUDE TEST MATHEMATICS SCORE AND HIGH SCHOOL RANK (N = 412).

HIGH SCHOOL RANK	FINAL GRADE	SAT MATH 200-499 f	cf	cp	500-549 f	cf	cp	550-800 f	cf	cp
	A	3	3	15	5	5	25	42	42	48
	B	8	11	55	9	14	70	30	72	83
1	C	3	14	70	5	19	95	10	82	94
	D	5	19	95	1	20	100	4	86	99
	F	1	20	100	0	20	100	1	87	100
	A	2	2	7	6	6	14	21	21	23
	B	2	4	15	17	23	53	31	52	57
2-3	C	12	16	59	11	34	79	26	78	85
	D	8	24	89	7	41	95	12	90	98
	F	3	27	100	2	43	100	2	92	100
	A	0	0	0	3	3	10	4	4	8
	B	3	3	7	7	10	33	5	9	18
4-10	C	8	11	26	8	18	60	23	32	63
	D	19	39	71	7	25	83	12	44	86
	F	12	42	100	5	30	100	7	51	100

TABLE 16. EXPECTANCY TABLE FROM STUDY III FOR FINAL GRADE IN MATHEMATICS M118 BASED ON CEEB SCHOLASTIC APTITUDE TEST MATHEMATICS SCORE AND HIGH SCHOOL RANK (N = 297).

HIGH SCHOOL RANK	FINAL GRADE	SAT MATH 200-499 f	200-499 cf	200-499 cp	500-549 f	500-549 cf	500-549 cp	550-800 f	550-800 cf	550-800 cp
1	A	1	1	*	1	1	10	28	28	38
	B	1	2	*	2	3	30	21	49	67
	C	0	2	*	5	8	80	19	68	93
	D	2	4	*	2	10	100	5	73	100
	F	1	5	*	0	10	100	0	73	100
2-3	A	1	1	6	1	1	4	16	16	17
	B	3	4	22	6	7	29	30	46	49
	C	5	9	50	13	20	83	34	80	86
	D	7	16	89	2	22	92	11	91	98
	F	2	18	100	2	24	100	2	93	100
4-10	A	0	0	0	0	0	0	6	6	17
	B	0	0	0	3	3	18	10	16	44
	C	7	7	33	7	10	59	9	25	69
	D	12	19	90	5	15	88	8	33	92
	F	2	21	100	2	17	100	3	36	100

*cf_T < 10 so no cps have been computed.

TABLE 17. EXPECTANCY TABLE FROM STUDY III FOR FINAL GRADE IN MATHEMATICS M119 BASED ON CEEB SCHOLASTIC APTITUDE TEST MATHEMATICS SCORE AND HIGH SCHOOL RANK (N = 149).

HIGH SCHOOL RANK	FINAL GRADE	SAT MATH 200-499 f	cf	cp	500-800 f	cf	cp
1	A	0	0	*	26	26	40
	B	2	2	*	19	45	69
	C	0	2	*	14	59	91
	D	1	3	*	2	61	94
	F	0	3	*	4	65	100
2-10	A	0	0	*	10	10	14
	B	3	3	*	24	34	47
	C	4	7	*	26	60	82
	D	0	7	*	9	69	95
	F	1	8	*	4	73	100

*cf_T < 10 so no cps have been computed.

TABLE 18. EXPECTANCY TABLE FROM STUDY III FOR FINAL GRADE IN MATHEMATICS M131 BASED ON CEEB SCHOLASTIC APTITUDE TEST MATHEMATICS SCORE AND HIGH SCHOOL RANK (N = 107).

HIGH SCHOOL RANK	FINAL GRADE	SAT MATH 200-499 f	cf	cp	SAT MATH 500-800 f	cf	cp
1	A	0	0	*	11	11	28
	B	1	1	*	18	29	73
	C	2	3	*	9	38	95
	D	0	3	*	2	40	100
	F	0	3	*	0	40	100
2-10	A	1	1	*	7	7	13
	B	1	2	*	14	21	38
	C	1	3	*	19	40	73
	D	4	7	*	10	50	91
	F	2	9	*	5	55	100

*$cf_T < 10$ so no cps have been computed.

TABLE 19. EXPECTANCY TABLE FROM STUDY III FOR FINAL GRADE IN MATHEMATICS M215 BASED ON CEEB SCHOLASTIC APTITUDE TEST MATHEMATICS SCORE AND HIGH SCHOOL RANK (N = 448).

HIGH SCHOOL RANK	FINAL GRADE	SAT MATH 200-499 f	cf	cp	500-549 f	cf	cp	550-800 f	cf	cp
1	A	0	0	*	1	1	6	58	58	31
	B	0	0	*	11	12	67	66	124	67
	C	1	1	*	4	16	89	44	168	91
	D	0	1	*	2	18	100	12	180	97
	F	0	1	*	0	18	100	5	185	100
2-3	A	0	0	*	3	3	15	17	17	18
	B	2	2	*	2	5	25	22	39	41
	C	2	4	*	7	12	60	36	75	78
	D	1	5	*	5	17	85	10	85	89
	F	2	7	*	3	20	100	11	96	100
4-10	A	0	0	0	0	0	0	9	9	10
	B	3	3	27	3	3	18	14	23	25
	C	2	5	45	9	12	71	41	64	69
	D	6	11	100	3	15	88	13	77	83
	F	0	11	100	2	17	100	16	93	100

*cf_T < 10 so no cps have been computed.

TABLE 20. EXPECTANCY TABLE FROM STUDY III FOR FINAL GRADE IN MATHEMATICS M216 BASED ON CEEB SCHOLASTIC APTITUDE TEST MATHEMATICS SCORE AND HIGH SCHOOL RANK (N = 238).

HIGH SCHOOL RANK	FINAL GRADE	SAT MATH 200-549 f	cf	cp	550-800 f	cf	cp
1	A	1	1	*	37	37	31
	B	1	2	*	36	73	61
	C	4	6	*	29	102	86
	D	2	8	*	12	114	96
	F	0	8	*	5	119	100
2-10	A	8	8	27	11	11	14
	B	7	15	50	19	30	37
	C	9	24	80	31	61	75
	D	3	27	90	12	73	90
	F	3	30	100	8	81	100

*cf_T < 10 so no cps have been computed.